

S-dual Inflation: BICEP2 data without unlikeliness

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We show that an S -dual inflationary potential solves the unlikeliness problem and explains the excess B -mode power observed by the BICEP2 experiment as arising from primordial tensor fluctuations.

The discovery of primordial gravitational waves in the B -mode power spectrum is fertile testing ground for inflationary models [1]. The B -mode power spectrum recently reported by the BICEP2 Collaboration is well-fit by a lensed- Λ CDM + tensor model, with a tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ and is inconsistent with the null hypothesis, $r = 0$, at a significance of $6 - 7\sigma$ depending on the modeling of foreground dust. Furthermore, the new BICEP2 data when combined with observations from the Wilkinson Microwave Anisotropy Probe (WMAP) [2], the Atacama Cosmology Telescope (ACT) [3], the South Pole Telescope (STP) [4], and the Planck mission [5] significantly shrink the space of allowed inflationary cosmologies.

Of particular interest here is that Planck data favor standard slow-roll single field inflationary models with plateau-like potentials $V(\phi)$ for which $V'' < 0$, over power-law potentials. However, most of these plateau-like inflaton potentials experience the so-called “unlikeliness problem” [6]. As an illustration of this problem consider the simplest example of a plateau-like potential,

$$V(\phi) = \frac{V_0}{(2M)^4} [\phi^2 - (2M)^2]^2, \quad (1)$$

where V_0 and M are free parameters. It is seen by inspection that the plateau terminates at a local minimum, and then for large ϕ , the potential grows as a power-law $\sim V_0\phi^4/(2M)^4$. A problem arises because the minimum of the potential can be reached in two different ways: by slow-roll along the plateau or by slow-roll from the power-law side of the minimum. The path from the power-law side requires less fine tuning of parameters, has inflation occurring over a much wider range of ϕ , and produces exponentially more inflation. Yet the data prefer the unlikely path along the plateau.

The requirement that $V'' < 0$ in the de Sitter region, and the avoidance of the unlikeliness problem, must now also accommodate the tensor-to-scalar ratio detected by

BICEP2 data. Also, we would like the inflaton potential to possess some connection to particle physics. To this end, we hypothesize that the potential be invariant under the S -duality constraint $g \rightarrow 1/g$, or $\phi \rightarrow -\phi$, where ϕ is the dilaton/inflaton, and $g \sim e^{\phi/M}$. Here M is expected to be within a few orders of magnitude of the Planck mass $M_{\text{Pl}} = G^{-1/2}$. (Throughout we use natural units, $c = \hbar \equiv 1$.) This requirement forces the functional form $V(\phi) = f[\cosh(\phi/M)]$ on the potential. In what follows we take for V the simplest S self-dual form

$$V = V_0 \operatorname{sech}(\phi/M), \quad (2)$$

which solves the unlikeliness problem because it has no power-law wall.

S duality had its origins in the Dirac quantization condition on the electric and magnetic charges, suggesting an equivalence in the description of quantum electrodynamics as either a weakly coupled theory of electric charges or a strongly coupled theory of magnetic monopoles. This was developed in [7] and extended into the S -dualities of Type IIB string theories [8]. We don't attempt a full association with a particular string vacuum, but simply regard the self-dual constraint as a relic of string physics in big bang cosmology.

For the potential in (2), the usual slow-roll parameters are [9]

$$\epsilon \equiv \frac{M_{\text{Pl}}^2}{16\pi} \left(\frac{V'}{V} \right)^2 = \frac{M_{\text{Pl}}^2 \tanh^2(\phi/M)}{16M^2\pi}, \quad (3)$$

and

$$\begin{aligned} \eta &\equiv \frac{M_{\text{Pl}}^2}{8\pi} \left| \frac{V''}{V} - \frac{1}{2} \left(\frac{V'}{V} \right)^2 \right| \\ &= \frac{M_{\text{Pl}}^2 | -5 + \cosh(2\phi/M) |}{32M^2\pi \cosh^2(\phi/M)}. \end{aligned} \quad (4)$$

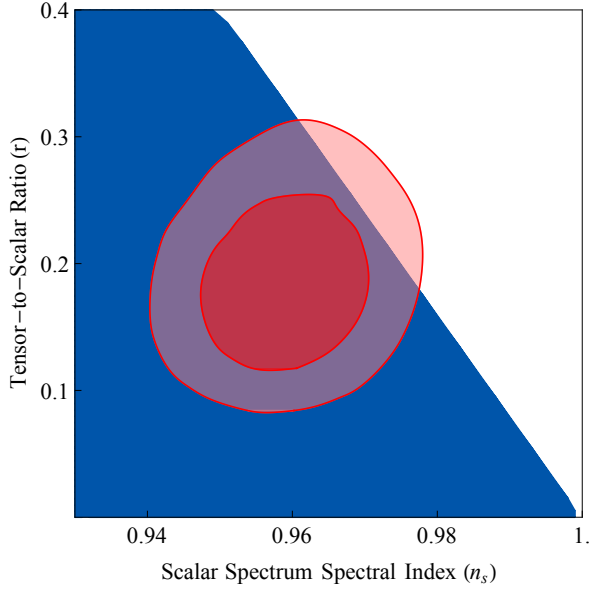


FIG. 1: The blue region shows the parameter space available to our potential. The red regions shows the 68% and 95% C.L. regions favored by a combination of Planck data, WMAP polarization data and small scale CMB data in combination with the BICEP2 constraint on r [1].

The density fluctuation at scale k is

$$\begin{aligned} \left. \frac{\delta\rho}{\rho} \right|_k &= \frac{H^2/2\pi}{(\dot{\phi}_k)_{\text{f.o.}}} \\ &= \frac{(8\pi)^{3/2}}{2\pi\sqrt{3}} \frac{1}{M_{\text{Pl}}^3} \left(\frac{V^{3/2}}{V'} \right)_k \Big|_{\text{f.o.}} \\ &= - \frac{8M\sqrt{2\pi/3} \coth(\phi/M) \sqrt{V_0/\cosh(\phi/M)}}{M_{\text{Pl}}^3} \Big|_{\text{f.o.}}, \end{aligned} \quad (5)$$

where $H = \dot{a}/a$ is the Hubble parameter and the field ϕ is evaluated at freeze out (f.o.), the time when the scale k leaves the horizon.

A straightforward calculation shows that (1) and (2) share similar behavior, namely that ϵ and η are of the scale M_{Pl}^2/M^2 . For (1) ϵ and η quickly grow near the end of inflation ($\phi \sim M$), whereas for (2) ϵ and η remain small. Thus for (2), as for power-law inflation (with an

exponential potential), inflation does not end. We assume that the dynamics of a second field leads to exit from the inflationary phase into the reheating phase. The requirement that there be roughly 60 e-folds of observable inflation is found to be

$$N \sim 8\pi(M/M_{\text{Pl}})^2 \gtrsim 60 \Rightarrow M \gtrsim 1.5 M_{\text{Pl}}. \quad (6)$$

The normalizations of the spectra are given by

$$A_s \simeq \frac{8V}{3M_{\text{Pl}}^4\epsilon}, \quad A_t \simeq \frac{128V}{3M_{\text{Pl}}^4}, \quad (7)$$

and the ratio of the amplitudes of the spectra at the pivot $k = k_*$, is

$$r \equiv \frac{A_t}{A_s} \simeq 16\epsilon. \quad (8)$$

Under the approximations of Ref. [10], the spectral indices and their running are

$$n_s \simeq 1 - 4\epsilon + 2\eta, \quad (9)$$

$$n_t \simeq -2\epsilon, \quad (10)$$

$$\frac{dn_s}{d\ln k} \simeq \frac{dn_t}{d\ln k} \simeq -4\epsilon(\epsilon - \eta). \quad (11)$$

The parameters M and ϕ_* (the field value when the k_* scale crosses the horizon, $k_* = aH$), are determined from the slow-roll parameters, which in turn are determined by r and n_s . V_0 is fixed by the density contrast $\delta\rho/\rho \sim 10^{-5}$. The parameter space that admits positive solutions for ϕ_* and M is shown in Fig. 1. With ϵ and η known, we also compute n_t , and the running of the spectral indices. In Table I, we display the parameters of the model, n_t and $dn/d\ln k$ for values of n_s and r within their 1σ uncertainties. The values of M agree with (6) modulo logarithmic corrections from the high field contributions to N .

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TABLE I: Model parameters, n_t and $dn/d\ln k$ for n_s and r within their 1σ uncertainties.

n_s	r	ϕ_*/M_{Pl}	M/M_{Pl}	V_0/M_{Pl}^4	$V(\phi_*)/M_{\text{Pl}}^4$	n_t	$dn_s/d\ln k \simeq dn_t/d\ln k$
0.95	0.20	1.18	1.03	1.80×10^{-11}	1.04×10^{-11}	-0.0255	-6.25×10^{-4}
0.95	0.27	1.42	0.974	3.19×10^{-11}	1.40×10^{-11}	-0.0345	-5.48×10^{-4}
0.95	0.15	1.02	1.08	1.16×10^{-11}	7.80×10^{-12}	-0.0191	-5.86×10^{-4}
0.96	0.20	1.51	1.11	2.17×10^{-11}	1.04×10^{-11}	-0.0254	-3.75×10^{-4}
0.96	0.27	1.98	1.04	4.83×10^{-11}	1.40×10^{-11}	-0.0344	-2.11×10^{-4}
0.96	0.15	1.28	1.16	1.30×10^{-11}	7.80×10^{-12}	-0.0190	-3.98×10^{-4}
0.97	0.20	2.25	1.20	3.45×10^{-11}	1.04×10^{-11}	-0.0253	-1.25×10^{-4}
0.97	0.15	1.74	1.28	1.62×10^{-11}	7.80×10^{-12}	-0.0190	-2.11×10^{-4}

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